# القيم العظمى والصغرى

maximum, minmum values

$f_x = 0 \qquad f_y = 0$

حل المعادلتين معاً نحصل على ⟵ critical point

النقط الحرجة

$$\Delta = f_{xx} \cdot f_{yy} - (f_{xy})^2$$

$\Delta > 0$ ⟶ $f_{xx} > 0$ minmum point ; $f_{xx} < 0$ max.

نقطة انقلاب

$\Delta < 0$ Saddle point

$\Delta = 0$ لا يوجد

⟵ اختبر القيم القصوى

**Ex** Find extrema value

$$f(x,y) = x^3 - 12x + y^2 + 4y - 6$$



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$f_x = 3x^2 - 12 \Rightarrow 0$ $\quad x = \pm 2$

$f_y = 2y + 4$ $\quad y = -2$

critical point $\underline{(2,-2)}$ , $\underline{(-2,-2)}$

$f_{xx} = 6x$ $\quad f_{yy} = 2$ $\quad f_{xy} = 0$

$\Delta = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= (6x)(2) = \underline{12x}$

$\underline{at (2,-2)}$ $\quad \Delta = 24 > 0$

$f_{xx} = 6x \Big|_{x=2} = 12 > 0$

$(2,-2)$ mini point

$\underline{at (-2,-2)}$ $\quad \Delta = 12x \Big|_{x=-2} = -24 < 0$

$(-2,-2)$ saddle point



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Determine extrema of function

$f(x,y) = 5x^2 - 3xy + y^2 - 15x - y + 2$ ①

$f_x = 10x - 3y - 15 = 0$

$f_y = -3x + 2y - 1 = 0$ نحل المعادلتين معاً

$+ \; 20x - 6y - 30 = 0$

$-9x + 6y - 3 = 0$

$11x - 33 = 0$

$x = 3$

$y = 5$

critical point $(3,5)$

$f_{xx} = 10$ $f_{yy} = 2$ $f_{xy} = -3$

$\Delta = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= 20 - 9 = 11 > 0$

$\because f_{xx} > 0$

$\therefore (3,5)$ local min

لإيجاد min value نعوض في ① عن

$x = 3$

$y = 5$

$f(3,5) =$ ______ ⑪

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Examine $f(x,y) = \sin x + \sin y + \sin(x+y)$

$0 < x < \pi/2$ , $0 < y < \pi/2$

for maxmum and minmum

$f_x = \cos x + \cos(x+y) = 0$ ①

$f_y = \cos y + \cos(x+y) = 0$

بالطرح

$\cos x - \cos y = 0$

$\cos x = \cos y$ $\qquad x = y$

بالتعويض في ①

$\cos x + \cos 2x = 0$

$\cos x + 2\cos^2 x - 1 = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = 1/2 \Longrightarrow \qquad x = \pi/3$

$\cos x = -1 \qquad x = \pi$

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$\therefore x = y$

$\therefore x = \pi/3 \longrightarrow y = \pi/3$

$x = \pi \longrightarrow y = \pi$

critical point $[\pi/3, \pi/3]$ , $[\pi, \pi]$ مرفوض

$f_{xx} = -\sin x - \sin(x+y) = -\sqrt{3}$

$f_{yy} = -\sin y - \sin(x+y) = -\sqrt{3}$

$f_{xy} = -\sin(x+y) = -\sqrt{3}/2$

$\Delta = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= (-\sqrt{3})(-\sqrt{3}) - 3/4 = 2.25 > 0$

$\therefore \Delta > 0, \quad f_{xx} = -\sqrt{3} < 0$

$[\pi/3, \pi/3)$ maxmum point

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Examin for max, min point

$$f(x,y) = xy(9-x-y)$$

الى

$$f(x,y) = 9xy - x^2y - xy^2$$

$$f_x = 9y - 2xy - y^2 = 0$$

$$f_y = 9x - 2xy - x^2 = 0 \quad (1)$$

بالطرح

$$9y - 9x - (y^2 - x^2) = 0$$

$$= 9(y-x) - (y-x)(y+x) = 0$$

$$(y-x)[9 - y - x] = 0$$

$y = x$ $\qquad\qquad$ $y = 9 - x$

if $y = x$ $\qquad$ بالتعويض م

$$9x - 2x^2 - x^2 = 0$$

$$9x - 3x^2 = 0 \Rightarrow x(9-3x) = 0$$

$x = 0$ $\qquad$ $x = 9/3$

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$y = x$

at $x = 0$ $\quad y = 0$

at $x = a/3$ $\quad y = a/3$

$[0, 0]$ و $[a/3, a/3]$

at $y = a - x$ بالتعويض

$ax - 2x(a - x) - x^2 = 0$

$x[a - 2a + 2x - x] = 0$

$x[-a + x) = 0$

$x = 0$ $\quad x = a$

$y = a - x$

at $x = 0$ $\quad y = a$

at $x = a$ $\quad y = 0$

$\therefore$ Critical Point $[0, 0], (a, 0), (0, a)$

$(\frac{a}{3}, a/3)$

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$f_{xx} = -2y \qquad f_{yy} = -2x$

$f_{xy} = a - 2x - 2y$

$\Delta = f_{xx} \, f_{yy} - (f_{xy})^2$

$= 4xy - [a - 2x - 2y]^2 \longrightarrow ②$

at $(0,0)$ $\quad \Delta = 0 - (a)^2 = -a^2 < 0$

$(0,0)$ Sadle Point

بالتعويض ②

at $(a,0)$

$\Delta = 0 - [a - 2a - 0]^2 = -a^2 < 0$

$(a,0)$ Sadle point

at $(0,a)$ $\quad \Delta = 0 - [a - 0 - 2a]^2 = -a^2 < 0$

$(0,a)$ Sadle Point

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بالتعويض ②

at $\left(\frac{a}{3}, a/3\right)$

$$\Delta = 4 \cdot \frac{a}{3} \cdot \frac{a}{3} - \left[a - \frac{2a}{3} - \frac{2a}{3}\right]^2$$

$$= \frac{4a^2}{9} - \frac{a^2}{9} = \frac{3a^2}{9} = \frac{a^2}{3} > 0$$

$f_{xx} = -2y$

$f_{xx} = -2a/3$

$\therefore \Delta > 0, \quad f_{xx} = -\frac{2a}{3}$

$a > 0 \quad f_{xx} < 0$

$\left(\frac{a}{3}, \frac{a}{3}\right)$

maxmum

$a < 0$

$\therefore f_{xx} > 0$

$\left(\frac{a}{3}, \frac{a}{3}\right)$

minmum

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Examin $f(x,y) = x^3 + y^3 - 3xy$ for maximum and minmum values

$f_x = 3x^2 - 3y = 0$

$f_y = -3y^2 + 3x = 0$ ①

بالجمع :

$3(x^2 - y^2) + 3(x - y) = 0$

$(x - y)(x + y + 1) = 0$

$x = -y - 1$

$x = y$

بالتعويض ②

at $x = y$

$3y^2 - 3y = 0$

$3y(y - 1) = 0$

$y = 0 \qquad y = 1$

$x = 0 \qquad x = 1$

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at $x = -y - 1$

بالتعويض ه

$\cancel{3} y^2 - \cancel{3}(-y - 1) = 0$

$y^2 + y + 1 = 0$ ليس لها حل

critical point $[0,0]$ , $[1,1]$

$f_{xx} = 6x$ $f_{yy} = 6y$

$f_{xy} = -3$

$\Delta = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= 36xy - 9$

at $(0,0)$ $\Delta = -9 < 0$

sadle point

at $(1,1)$ $\Delta = 36 - 9 = 27 > 0$

$\Delta > 0$ $f_{xx} = 6x \big|_{x=1} = 6 > 0$

$(1,1)$ min Point

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